UNIFORM BOUNDS OF BASE CHANGE CONDUCTORS AND LINK WITH THE GENERALIZED SZPIRO CONJECTURE

HUAJUN LU

ABSTRACT. The difference between the Faltings height of an abelian variety A defined over a number field k and its stable height is measured by the so-called base change conductor. In this paper, we give a uniform bound of the base change conductor in terms of the dimension of A and the degree of k. This allows us to reduce the generalized Szpiro conjecture to the semi-stable case.

1. INTRODUCTION

Let A be an abelian variety of dimension g defined over a number field k of degree d. Let $\mathcal{O}_k$ be the ring of integers of k and let $\mathcal{A}$ be the Néron model of A over Spec $\mathcal{O}_k$. Let $\epsilon : \text{Spec } \mathcal{O}_k \to \mathcal{A}$ be the zero section of $\mathcal{A}$ and let $\Omega^1_{\mathcal{A}}$ be the sheaf of relative differentials of $\mathcal{A}$ over $\mathcal{O}_k$. A global section ω of $\Omega^1_{\mathcal{A}}$ can be seen as a top degree differential form which is invariant under the translations on A. The *Faltings height* of A is defined by

$$h_{Fal}(A) = \frac{1}{[k:\mathbb{Q}]}(\log \#(\Omega/\omega\mathcal{O}_k) - \sum_{\sigma:k\hookrightarrow\mathbb{C}} \log ||\omega||_\sigma),$$

where $\Omega = \Gamma(\text{Spec } \mathcal{O}_K, \epsilon^* \wedge^g \Omega^1_{\mathcal{A}})$, σ runs over all embeddings of k in $\mathbb{C}$ and the norm of ω with respect to σ is defined as in [Fal83]:

$$||\omega||^2_\sigma = \left(\frac{\sqrt{-1}}{2}\right)^g \int_{A_\sigma(\mathbb{C})} \omega \wedge \overline{\omega}, \quad \forall\ \sigma : k \hookrightarrow \mathbb{C}.$$

The Faltings height of A does not depend on the choice of ω by the product formula of algebraic numbers.

A central problem on the Faltings height is the generalized Szpiro conjecture, which claims the following: Given a number field k and an integer g, there are constants $c_1(k,g)$ and $c_2(k,g)$, depending only on k and g, such that for every abelian variety A over k of dimension g, the Faltings height $h_{Fal}(A)$ satisfies

$$h_{Fal}(A) \leq c_1(k,g) \log N_{k/\mathbb{Q}}(\mathcal{F}_{A/k}) + c_2(k,g)$$

Key words and phrases. Faltings height, base change conductors, abelian varieties, uniform bounds.
MSC 2010: 11G10, 11G50, 14G40.

where $\mathcal{F}_{A/k}$ is the conductor of A [BK94] and $N_{k/\mathbb{Q}}$ denotes the absolute norm map of $k/\mathbb{Q}$.

To help study the Faltings height, a closed related notion is that of *stable height*. It is defined as follows. By the semi-stable reduction theorem of Grothendieck, there exists a finite field extension l/k such that $A_l = A \times_{\mathrm{Spec}\ k} \mathrm{Spec}\, l$ has semi-stable reduction. The Faltings height $h_{Fal}(A_l)$ does not depend on the choice of l and is called the *stable height* of A. Let P be the set of primes of $\mathcal{O}_k$ at which A does not have semi-stable reduction. For each prime $\mathfrak{p}$ of $\mathcal{O}_k$, let $N\mathfrak{p}$ be the cardinality of the residue field of $\mathcal{O}_k$ at $\mathfrak{p}$. Then we have

$$h_{Fal}(A_l) - h_{Fal}(A) = \frac{-1}{[k:\mathbb{Q}]} \sum_{\mathfrak{p}\in P} c(A,\mathfrak{p}) \log(N\mathfrak{p})$$

for some positive rational number $c(A,\mathfrak{p}), \mathfrak{p} \in P$ uniquely determined by A and $\mathfrak{p}$. Following Chai and Yu ([Chai00] and [CY01]), we will refer to the numerical invariants $(c(A,\mathfrak{p}))_{\mathfrak{p}\in P}$ as the *base change conductor* of A. We will also call $c(A,\mathfrak{p})$ the local base change conductor of A at $\mathfrak{p}$.

In view of the generalized Szpiro conjecture, a natural question is to ask whether the local base change conductor $c(A,\mathfrak{p})$ is uniformly bounded. In this paper, we give a positive answer to this question. More precisely, our main result is the following.

Theorem 1.1. *There is a function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ having the following property: For every number field k of degree $d = [k:\mathbb{Q}]$ and every abelian variety A of dimension g over k, the local base change conductors $c(A,\mathfrak{p})$ all satisfy $c(A,\mathfrak{p}) \le f(d,g)$.*

In fact, our proof will give an explicit construction of a possible choice for f. As an application, we will show that the generalized Szpiro conjecture can be reduced to the semi-stable case (cf. Theorem 3.4).

The proof of our main theorem makes use of some local analysis on Lie algebras and Weil restrictions as well as a theorem of Raynaud. A key step is to prove a bound for the base change conductor of a Weil restriction in the local case.

2. Some reviews on Weil restriction

Let $\pi : S' \to S$ be a finite and locally free morphism of schemes. Let X' be a quasi-projective S'-scheme. Then the functor $R_{S'/S}(X')$ from the category of S-schemes to the category of sets defined by

$$R_{S'/S}(X') : Sch/S \to (Sets); \qquad T \mapsto \mathrm{Hom}_{S'}(T \times_S S', X')$$

is representable by a quasi-projective S-scheme (cf. [BLR90], §7.6, Theorem 4), called the *Weil restriction* of X' with respect to π. By abuse of notation, we also denote by $R_{S'/S}(X')$ this representing scheme. If there is no confusion about π, we call it the Weil restriction of X' for short. It is clear

that if X' is a group scheme over S', then $R_{S'/S}(X')$ is a group scheme over S.

If $\pi : S' \to S$ is a finite and flat morphism of connected Dedekind schemes, Néron models behave well with respect to Weil restrictions. More precisely, let K and K' be the function fields of S and S' respectively and let $\mathcal{A}' \to S'$ be an abelian scheme which is the Néron model of its generic fiber $A' = \mathcal{A}' \times_{S'} \mathrm{Spec} K'$. Then by [BLR90], §7.6, Proposition 6, the Weil restriction $\mathcal{A} = R_{S'/S}(\mathcal{A}')$ exists and is the Néron model of its generic fiber $\mathcal{A} \times_S \mathrm{Spec}\ K = R_{K'/K}(A')$.

Example 2.1 Let $S' = \mathrm{Spec}\ L$ and $S = \mathrm{Spec}\ K$ where L/K is a finite separable field extension. Let X be a quasi-projective K-scheme. For later use, let us recall the description of the Weil restriction $Y = R_{L/K}(X_L)$ of X_L via Galois descent.

Let F/K be a finite Galois extension with Galois group Γ and assume there is a K-embedding $L \hookrightarrow F$. Let Λ be the set of K-embeddings of L in F. To each $\tau \in \Lambda$, we associate an F-scheme

$$X'_\tau = X_L \times_{(L,\tau)} \mathrm{Spec} F = X \times_K \mathrm{Spec}\ \ F = X_F.$$

The schemes $X'_\tau, \tau \in \Lambda$ are all the same as F-schemes but they have different L-scheme structures. Let $Y' = \prod_{\tau\in\Lambda} X'_\tau$ be the fiber product of the X'_τ's over Spec F.

Let us assume that $X = \mathrm{Spec}\ A$ is affine. Then we can define a semi-linear Γ-action on Y' by

$$\gamma(\underset{\tau\in\Lambda}{\otimes} a_\tau) = \underset{\tau\in\Lambda}{\otimes} a'_\tau \text{ with } a'_{\gamma\tau} = (id \otimes \gamma)(a_\tau)$$

for any $\gamma \in \Gamma$ and $a_\tau = a \otimes x \in A \otimes_K F = A'_\tau$ with $a \in A$ and $x \in F$. Let $Y = Y'/\Gamma$ be the quotient of Y' by Γ. Then Y is the Weil restriction of X_L by the theory of Galois descent ([KMRT98], Page 329).

In general, since X is separated and quasi-compact, we can find a finite affine open covering $\{U_i, i = 1, ..., n\}$ of X such that $U_{ij} = U_i \cap U_j$ are also affine. Then the Weil restriction $R_{L/K}(X_L)$ of X_L can be obtained by gluing $R_{L/K}(U_i \times_K \mathrm{Spec}\ L)$ along $R_{L/K}(U_{ij} \times_K \mathrm{Spec}\ L)$.

3. Uniform bounds of base change conductors

We shall prove our main results in this section. To start with, let us recall the definition of base change conductors.

Let $K = k_\mathfrak{p}$ be the completion of a number field k at a finite prime $\mathfrak{p}$. Let A be an abelian variety of dimension g over K with Néron model $\mathcal{A}$ over the ring of integers $\mathcal{O}_K$ of K. Let L/K be a finite field extension such that A_L has semi-stable reduction. Let $\mathcal{A}'$ be the Néron model of A_L over the ring of integers $\mathcal{O}_L$ of L. By the universal property of Néron model, there exists a unique morphism $\phi : \mathcal{A} \times \mathrm{Spec}\ \mathcal{O}_L \longrightarrow \mathcal{A}'$ which extends the canonical isomorphism between the generic fibers. Let ϵ denote the zero section of

the abelian scheme $\mathcal{A}/\mathcal{O}_K$ or $\mathcal{A}'/\mathcal{O}_L$ and let $\Omega^1_{\mathcal{A}}$ (resp. $\Omega^1_{\mathcal{A}'}$) be the sheaf of relative differentials of $\mathcal{A}/\mathcal{O}_K$ (resp. $\mathcal{A}'/\mathcal{O}_L$). Then the quotient

$$\frac{\Gamma(\mathrm{Spec}\ \mathcal{O}_K,\ \epsilon^* \wedge^g \Omega^1_{\mathcal{A}}) \otimes \mathcal{O}_L}{\Gamma(\mathrm{Spec}\ \mathcal{O}_L,\ \epsilon^* \wedge^g \Omega^1_{\mathcal{A}'})}$$

is an $\mathcal{O}_L$-module of finite length. We define

$$c(A) = \frac{1}{e(L/k_{\mathfrak{p}})} \mathrm{length}_{\mathcal{O}_L} \left(\frac{\Gamma(\mathrm{Spec}\ \mathcal{O}_K,\ \epsilon^* \wedge^g \Omega^1_{\mathcal{A}}) \otimes \mathcal{O}_L}{\Gamma(\mathrm{Spec}\ \mathcal{O}_L,\ \epsilon^* \wedge^g \Omega^1_{\mathcal{A}'})} \right)$$

where $e(L/K)$ is the ramification index of L/K. One verifies that the number $c(A)$ is independent of the choice of L.

Note that the module $\Gamma(\mathrm{Spec}\ \mathcal{O}_K, \epsilon^*\Omega^1_{\mathcal{A}})$ may be viewed as the module of the translation invariant differential forms on $\mathcal{A}$ and is dual to the Lie algebra $Lie(\mathcal{A})$ of $\mathcal{A}$. Hence we can rewrite

$$c(A) := \frac{1}{e(L/K)} \mathrm{length}_{\mathcal{O}_L} \frac{Lie(\mathcal{A}')}{Lie(\mathcal{A}) \otimes \mathcal{O}_L}.$$

Proposition 3.1. *With notation as above, assume that L/K is a Galois extension and put $B = R_{L/K}(A_L)$. Then we have*

$$c(A) \leq c(B) \leq \delta_{L/K} \mathrm{dim} A,$$

where $\delta_{L/K}$ is the normalized K-valuation (i.e. the set of the values of the valuation is $\mathbb{Z}$) of the discriminant of L/K.

Proof. Let Γ be the Galois group of L/K and let F be another copy of L. Applying the result in Example 2.1 to $X = A$, we get $B_F = \prod_{\tau \in \Lambda} A_{F,\tau}$ and $Lie(B_F) = \bigoplus_{\tau \in \Lambda} Lie(A_{F,\tau})$. On the other side, by the functoriality of Lie algebras and Weil restrictions, we have $Lie(B_F) = Lie(B) \otimes_K F = Lie(A_L) \otimes_K F$. Hence we have an isomorphism

$$\Psi : Lie(A_L) \otimes_K F \to \bigoplus_{\tau \in \Lambda} Lie(A_{F,\tau}) \quad t \otimes x \mapsto (x\tau_*(t))_\tau, \quad t \in Lie(A_L), x \in F,$$

where $\tau_* : Lie(A_L) \to Lie(A_F)$ is induced by $\tau : L \to F$.

Let $\mathcal{O}_F$ be the ring of integers of F. Let $\mathcal{A}$ (resp. $\mathcal{B}, \mathcal{A}', \mathcal{B}'$) be the Néron model of A (resp. B, A_L, B_F) over $\mathcal{O}_K$ (resp. $\mathcal{O}_K, \mathcal{O}_L$, $\mathcal{O}_F$). Since $B_F = \prod_{\tau \in \Lambda} A_{F,\tau}$, we have $\mathcal{B}' = \prod_{\tau \in \Lambda} \mathcal{A}'_{\mathcal{O}_F,\tau}$ and $Lie(\mathcal{B}') = \bigoplus_{\tau \in \Lambda} Lie(\mathcal{A}'_{\mathcal{O}_F,\tau})$. By the universal property of Néron models, there is a morphism $\phi : \mathcal{B} \times_{\mathcal{O}_K} \mathrm{Spec}\ \mathcal{O}_F \to \mathcal{B}'$ extending the isomorphism of generic fibers. Then the morphism ϕ induces a morphism $\phi_{\mathcal{B}} : Lie(\mathcal{B}) \otimes_{\mathcal{O}_K} \mathcal{O}_F \to Lie(\mathcal{B}')$ which can be identified with the restriction of Ψ on $Lie(\mathcal{B}) \otimes_{\mathcal{O}_K} \mathcal{O}_F$. Note that we may identify Λ with Γ and write $\phi_{\mathcal{B}}$ as

$$\phi_{\mathcal{B}} : Lie(\mathcal{A}'_{\mathcal{O}_F}) \otimes_{\mathcal{O}_K} \mathcal{O}_F \to \bigoplus_{\gamma \in \Gamma} Lie(\mathcal{A}'_{\mathcal{O}_F,\gamma}) \quad t \otimes x \mapsto (\gamma_*(t)x)_\gamma.$$

Then the cokernel of $\phi_{\mathcal{B}}$ is killed by the different ideal $\mathfrak{D}_{\mathcal{O}_L/\mathcal{O}_K}$ of $\mathcal{O}_L$ over $\mathcal{O}_K$ by Proposition 3.3 of [LL] applied to $M = Lie(\mathcal{B}')$ and $L = F$.

Hence we have

$$c(B) = \frac{1}{e(L/K)}\mathrm{length}_{\mathcal{O}_L}\mathrm{Coker}\phi_{\mathcal{B}} \leq \frac{[L:K]\mathrm{dim}A}{e(L/K)}\mathrm{length}_{\mathcal{O}_L}\mathfrak{D}_{\mathcal{O}_L/\mathcal{O}_K} \leq \delta_{L/K}\mathrm{dim}A.$$

For each τ let p_τ be the canonical projection of $B_F = \Pi_{\tau\in\Lambda}A_{F,\tau}$ onto the τ-component and let $v = \sum_{\tau\in\Lambda} p_\tau$. Then v is Γ-invariant, hence $v : B \to A$ is defined over K. By considering the morphism between Néron models induced by v we get a commutative diagram of Lie algebras:

$$\begin{array}{ccc} Lie(\mathcal{B})\otimes\mathcal{O}_L & \longrightarrow & Lie(\mathcal{A})\otimes\mathcal{O}_L \\ \phi_{\mathcal{B}}\downarrow & & \phi_{\mathcal{A}}\downarrow \\ Lie(\mathcal{B}') & \xrightarrow{v_*} & Lie(\mathcal{A}'), \end{array}$$

where both horizontal morphisms are induced by v.

By Example 2.1, we have $Lie(\mathcal{B}') = \underset{\tau\in\Lambda}{\oplus} Lie(\mathcal{A}'_\tau)$ and $v_*((x_\tau)_\tau) = \sum_{\tau\in\Lambda} x_\tau$ for any $(x_\tau)_\tau \in \underset{\tau\in\Lambda}{\oplus} Lie(\mathcal{A}')_\tau$. Hence v_* is surjective and consequently the map from $\mathrm{Coker}(\phi_{\mathcal{B}})$ to $\mathrm{Coker}(\phi_{\mathcal{A}})$ induced by v_* is surjective. Since $c(B) = \frac{1}{e(L/K)}\mathrm{length}_{\mathcal{O}_L}$ $\mathrm{Coker}(\phi_{\mathcal{B}})$ and $c(A) = \frac{1}{e(L/K)}\mathrm{length}_{\mathcal{O}_L}$ $\mathrm{Coker}(\phi_{\mathcal{A}})$, we get $c(A) \leq c(B)$. This completes the proof of the proposition.

□

Remark 3.2 The bound given above is not sharp in general. When A has potential ordinary reduction, Chai [Chai00] proved that $\chi(A)/4 \leq c(A) \leq \chi(A)/2$ where $\chi(A)$ is the Artin conductor of A.

Now let A be an abelian variety over a number field k. Recall that the *base change conductor* of A/k is determined as follows: Let P be the set of primes of the ring of integers $\mathcal{O}_k$ of k at which A does not have semi-stable reduction. For each $\mathfrak{p} \in P$, let $c(A,\mathfrak{p})$ be the number $c(A_{k_\mathfrak{p}})$ defined earlier for the abelian variety $A_{k_\mathfrak{p}}$ over the local completion $k_\mathfrak{p}$. Then the base change conductor of A/k is $(c(A,\mathfrak{p}))_{\mathfrak{p}\in P}$.

We are now ready to prove our main result, Theorem 1.1, which states that $c(A,\mathfrak{p})$ can be uniformly bounded by a number $f(d,g)$ depending only on the degree d of the number field k and the dimension g of the abelian variety A.

Proof of Theorem 1.1. Let n be a positive integer and let $A[n]$ be the kernel of the multiplication-by- n map on A. Let $k(A[n])$ be the composition field of the residue fields of the points of $A[n]$. A theorem of Raynaud([SGA7], Proposition 4.7) asserts that if $n \geq 3$, then $A_{k(A[n])}$ has semi-stable reduction away from n. In particular, for $n = 15$ and $l = k(A[15])$, A_l has semi-stable reduction everywhere and we have

$$[l:k] \leq f_1(g) := |GL_{2g}(\mathbb{Z}/3\mathbb{Z})| * |GL_{2g}(\mathbb{Z}/5\mathbb{Z})|.$$

Let $\mathfrak{p}$ be a prime of k, $\mathfrak{P}$ a prime of l lying over k and let $G_\mathfrak{P}$ be the decomposition group of $\mathfrak{P}|\mathfrak{p}$. Then the ramification filtration of $G_\mathfrak{P}$ has length at most $v_{l_\mathfrak{P}}(p)/(p-1)$ ([Ser79], §IV.2, Exercise 3c) where $v_{l_\mathfrak{P}}$ is the

normalized $\mathfrak{P}$-valuation of $l_{\mathfrak{P}}$. Then the normalized $\mathfrak{P}$-adic valuation of the different of $l_{\mathfrak{P}}/k_{\mathfrak{p}}$ is smaller than $|G_{\mathfrak{P}}|v_{l_{\mathfrak{P}}}(p)/(p-1)$ by [Ser79], Chap IV, Proposition 4. So the local base change conductor $c(A,\mathfrak{p})$ of A at $\mathfrak{p}$ is smaller than $g|G_{\mathfrak{P}}|^2 v_{k_{\mathfrak{p}}}(p)/(p-1)$ by Proposition 3.1. Hence the function $f(d,g) := f_1^2(g)dg + 1$ has the desired property. $\square$

Proposition 3.3. *The generalized Szpiro conjecture is equivalent to the following statement:*

There exist positive functions $c_1(d,g)$, $c_2(d,g)$ and $c_3(d,g)$, in variables d and g, such that for every number field k of degree d and every abelian variety A/k of dimension g, one has

$$h_{Fal}(A) \le c_1(d,g)\log N_{k/\mathbb{Q}}(\mathcal{F}_{A/k}) + c_2(d,g)\log \Delta_{k/\mathbb{Q}} + c_3(d,g), \tag{1}$$

where $\mathcal{F}_{A/k}$ is the conductor of A and $\Delta_{k/\mathbb{Q}}$ is the discriminant of $k/\mathbb{Q}$.

Proof. We need only to show that the generalized Szpiro conjecture implies the inequality (1).

Let $B = R_{k/\mathbb{Q}}(A_k)$, then by Theorem 1.1 we have

$$h_{Fal}(B) \ge h_{Fal}(B_l) = d\ h_{Fal}(A_l) \ge d(h_{Fal}(A_k) - f(d,g)\log N_{k/\mathbb{Q}}(\mathcal{F}_{A/k})),$$

where $l = k(A[15])$, $f(d,g)$ is the same as in Theorem 1.1 and the equality $h_{Fal}(B_l) = d\ h_{Fal}(A_l)$ holds since $B_l \cong A_l^d$.

Hence we have

$$h_{Fal}(A) \le \frac{1}{d}h_{Fal}(B) + f(d,g)\log N_{k/\mathbb{Q}}(\mathcal{F}_{A/k}).$$

By the generalized Szpiro conjecture, there are constants $c_1'(dg), c_2'(dg)$ depending only on dg (and $\mathbb{Q}$) such that

$$h_{Fal}(B) \le c_1'(dg)\log \mathcal{F}_{B/\mathbb{Q}} + c_2'(dg).$$

By Proposition 1 in [Mil72], we have $\mathcal{F}_{B/\mathbb{Q}} = N_{k/\mathbb{Q}}(\mathcal{F}_{A/k})\Delta_{k/\mathbb{Q}}^d$. Combining the above inequalities proves the proposition. $\square$

Theorem 3.4. *The generalized Szpiro conjecture is true if and only if the inequality* (1) *holds for all abelian varieties A over k which have semi-stable reduction over k.*

Proof. We just need to show if the inequality (1) holds for all abelian varieties over k which have semi-stable reduction, then it holds for all abelian varieties (with possibly different choice of $c_i(d,g), i = 1,2,3$).

Let A be a g-dimension abelian variety over k and let $l = k(A[15])$. Since l/k is unramified outside the places above 3 and 5 and the degree $[l:k]$ is bounded by a function of g, $\log(\Delta_{l/\mathbb{Q}}) - \log(\Delta_{k/\mathbb{Q}})$ is bounded from above by a function in variables d and g. By Theorem 1.1, we have $h_{Fal}(A) \le h_{Fal}(A_l) + f(d,g)\log N_{k/\mathbb{Q}}(\mathcal{F}_{A/k})$. It then follows that $h_{Fal}(A)$ satisfies the inequality 1 provided that the same inequality holds for all abelian varieties with semi-stable reduction.

$\square$

Acknowledgements. I would like to express my gratitude to Professor Marc Hindry who kindly purposed the problem to me with useful suggestions. I also thank Doctor Yong Hu for his useful comments.

References

[BLR90] S. Bosch, W. Lütkebohmert and M. Raynaud, Néron Models, Ergeb. Math. Grenz., **21**, Springer-Verlag, New York (1990).

[BK94] A. Brumer, K. Kramer, *The conductor of an abelian variety*, Compositio Math. **92** (1994) No. 2, 227-248 .

[Chai00] C-L. Chai, *Néron models for semiabelian varieties: congruence and change of base field*, Asian J. Math. vol. **4** (2000), No. 4, 715-736.

[CY01] C-L. Chai and J-K. Yu, *Congruences of Néron models for tori and the Artin conductor*, With an appendix by E. de Shalit, Ann. of Math. **154** (2001), 347-382.

[SGA7] A. Grothendieck, *Modèles de Néron et monodromie*, in Groupes de Monodromie en Géometrie Algébrique, SGA7 I, A. Grothendieck, ed., Lecture Notes in Math. **288**, Springer, Berlin-Heidelberg-New York, 1972, 313-523.

[Fal83] G. Faltings, *Endlichkeitssätze für abelsche Varietäten über Zahlkörpern*, Invent. Math. **73** (1983), no. 3, 349-366.

[KMRT98] M. Knus, A. Merkurjev, M. Rost, and J-P. Tignol, The Book of Involutions, AMS Colloquium Publications, Vol. **44**, 1998.

[LL] Q. Liu, and H. LU, *Congruences of models of elliptic curves*, preprinted.

[Mil72] J.S. Milne, *On the Arithmetic of Abelian Varieties*, Invent. Math. **17** (1972). 177-190.

[Ser79] J-P. Serre, Local Fields, Grad. Texts in Math. **67**, Springer-Verlag, New York (1979).

Academy of Mathematics and Systems Science, Beijing, 100190, China
E-mail address: huajun@amss.ac.cn